Bose Einstein Scholarship Test

An endeavour of International Research Scholars and Mentors with JMMC Research Foundation

Sample Question for Class - 12

1. Let $f:(0,1)\to(0,1)$ be a differentiable function such that $f'(x)\neq 0$ for all $x\in(0,1)$ and $f\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$. Suppose for all $x, \lim_{t\to x}\left(\frac{\int_0^t\sqrt{1-(f(s))^2}\,ds-\int_0^x\sqrt{1-(f(s))^2}\,ds}{f(t)-f(x)}\right)=f(x)$. Then the value of $f\left(\frac{1}{4}\right)$ belongs to :

   (a) $\left\{\frac{\sqrt{7}}{4},\frac{\sqrt{15}}{4}\right\}$ (b) $\left\{\frac{\sqrt{7}}{3},\frac{\sqrt{15}}{3}\right\}$ (c) $\left\{\frac{\sqrt{7}}{2},\frac{\sqrt{15}}{2}\right\}$ (d) $\{\sqrt{7},\sqrt{15}\}$

2. Evaluate $\int\frac{\left(\sqrt[3]{x+\sqrt{2-x^2}}\right)\left(\sqrt[6]{1-x\sqrt{2-x^2}}\right)dx}{\sqrt[3]{1-x^2}}; x\in(0,1)$:

   (a) $2^{\frac{1}{6}}x+C$ (b) $2^{\frac{1}{12}}x+C$ (c) $2^{\frac{1}{3}}x+C$ (d) None of these

3. If $\int_0^1\left(\sum_{r=1}^{2013}\frac{x}{x^2+r^2}\right)\left(\prod_{r=1}^{2013}(x^2+r^2)\right)dx=\frac{1}{2}\left[\left(\prod_{r=1}^{2013}(1+r^2)\right)-k^2\right]$ then k =

   (a) 2013 (b) 2013! (c) 2013^2 (d) 2013^{2013}

4. Let $f(x)$ be differentiable function on the interval $(0,\infty)$ such that $f(1)=1$ and $\lim_{t\to x}\left(\frac{t^3f(x)-x^3f(t)}{t^2-x^2}\right)=\frac{1}{2}\forall x>0$, then $f(x)$ is

   (a) $\frac{1}{4x}+\frac{3x^2}{4}$ (b) $\frac{3}{4x}+\frac{x^3}{4}$ (c) $\frac{1}{4x}+\frac{3x^3}{4}$ (d) $\frac{1}{4x^3}+\frac{3x}{4}$

5. The quadratic polynomials defined on real coefficients $p(x)=a_1x^2+2b_1x+c_1, Q(x)=a_2x^2+2b_2x+c_2. P(x) and Q(x)$ both take positive values $\forall x\in R$. If $f(x)=a_1a_2x^2+b_1b_2x+c_1c_2$, then :

   (a) $f(x)<0\forall x\in R$ (b) $f(x)>0\forall x\in R$

   (c) $f(x)$ takes both positive and negative values (d) Nothing can be said about $f(x)$

6. Number of points at which the function $\begin{matrix}x\le 1 & if -\infty<x<1\\ x>1 & if x\ge 1\end{matrix}$ is not derivable is :

   (a) 0 (b) 1 (c) 2 (d) 3

7. $\lim_{n\to\infty}\sum_{k=1}^{n}\left(\sin\frac{\pi}{2k}-\cos\frac{\pi}{2k}-\sin\left(\frac{\pi}{2(k+2)}\right)+\cos\frac{\pi}{2(k+2)}\right)=$

   (a) 0 (b) 1 (c) 2 (d) 3

8. Range of $f(x)=\sqrt{\sin(\log_7(\cos(\sin x)))}$ is :

   (a) [0,1) (b) {0,1} (c) {0} (d) [1,7]

9. Let $f: R\to R$ is defined by $f(x)=\begin{cases}(x+1)^3 & ; x\le 1\\ \ln x+(b^2-3b+10) & ; x>1\end{cases}$. If $f(x)$ is invertible, then the set of all values of 'b' is:

   (a) {1,2} (b) ϕ (c) {2,5} (d) None of these